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Exercise Sheet 4

Nov 18, 2019

Please tell me in/after the lecture on Monday what you want to have discussed

Exercise 1. Determine the ring of integers in $L = \mathbb{Q}(\alpha)$, where α is a root of $X^3 + X + 1$.

Exercise 2. Let $L = \mathbb{Q}(\alpha)$ where α is a root of $X^3 - X^2 - 2X - 8$.

(a) Show that $\{1, \alpha, \frac{\alpha+\alpha^2}{2}\}$ is an integral basis of $\mathcal{O}_L$.

(b) Show that there is no $\beta \in L$ such that $\mathcal{O}_L = \mathbb{Z}[\beta]$, i.e. the maximal order is *not* an equation order.

Exercise 3. Let L be an algebraic number field. Show that $d_{\mathcal{O}_L} \equiv 0$ or $1 \bmod 4$. (*Hint:* Decompose the determinant into a sum over even and over odd permutations)

Exercise 4. Compute the Hermite and Smith normal form of the following matrices:

(a)

$$A = \begin{pmatrix} 2 & 6 & 9 \\ -2 & 0 & 4 \\ 2 & 1 & -1 \end{pmatrix} \in \mathrm{Mat}_3(\mathbb{Z}).$$

(b)

$$\begin{pmatrix} 0 & 0 & 2-X \\ 0 & 1+X & 2X \\ 2-X & 0 & 0 \end{pmatrix} \in \mathrm{Mat}_3(\mathbb{Q}[X]) \ .$$

Remember to decide on a system of non-associates and of residues in $\mathbb{Q}[X]$.

(c)

$$\begin{pmatrix} 2-i & 2 \\ 7-i & 3+i \end{pmatrix} \in \mathrm{Mat}_3(\mathbb{Z}[i]) \ .$$

Remember to decide on a system of non-associates and of residues in $\mathbb{Z}[i]$.