ERRATUM

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Abstract. I am collecting here corrections and comments to my publications. Luckily, there was nothing fatal yet.

8. Highest weight theory for finite-dimensional graded algebras with triangular decomposition[1]

For Theorem 5.1 to be valid we have to assume that A is *self-injective*, which is not mentioned in the paper. Below, we will give a corrected proof of the theorem under this assumption and a counter-example to the claim without it. Since we apply the theorem only in the self-injective situation, it's not dramatic.

The proof, as written in the paper, contains a mistake that was pointed out to us by Jonathan Brundan in February 2020 (many thanks for this!): in the middle of the proof, after diagram (68), we claim that $\mathrm{Ext}^1_{\mathcal{G}(A)}(M, \mathrm{Ker}\,\pi) = 0$. This claim is false since $\mathrm{Ker}\,\pi$ has a standard filtration, but we would require it to have a costandard filtration in order for our argument to work.

Under the assumption that A is self-injective we can fix the proof by starting with a *costandard* filtration of the tilting object M. Here are the details. We will need Theorem 5.8, which is independent of Theorem 5.1. So, we will move Theorem 5.1 and Corollary 5.7 after Theorem 5.8 and assume everywhere that A is self-injective.

Theorem 5.1* *If A is self-injective, the tilting objects in $\mathcal{G}(A)$ are precisely the projective objects.*

Proof. A projective object M is projective-injective and the same argument as in the proof of Theorem 5.1 shows that it is tilting.

Conversely, assume that M is tilting. We need to show that it is projective. Let $0 = M_0 \subset M_1 \subset \ldots \subset M_{s-1} \subset M_s = M$ be a *costandard* filtration with $M_i/M_{i-1} \simeq \nabla(\lambda_{i-1})$. Let $q\colon M \twoheadrightarrow \nabla(\lambda_{s-1})$ be the quotient morphism. Let $\lambda := \lambda_{s-1}^{h^{-1}}$, where h is the permutation from Theorem 5.8(a). We know from Theorem 5.8(e) that $\nabla(\lambda_{s-1})$ is at the top of every costandard filtration of $P(\lambda)$. We thus have a quotient morphism $\pi\colon P(\lambda) \twoheadrightarrow \nabla(\lambda_{s-1})$. Due to the projectivity of $P(\lambda)$

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[1]Adv. Math. 330 (2018) 361–419. With G. Bellamy.

there is $\eta\colon P(\lambda) \to M$ making the diagram

$$\begin{tikzcd} & M \arrow[d, two heads, "q"] \\ P(\lambda) \arrow[ur, dashed, "\eta"] \arrow[r, two heads, "\pi"'] & \nabla(\lambda_{s-1}) \end{tikzcd} \tag{68*}$$

commutative. Since $P(\lambda)$ is projective, it has a costandard filtration by the above, hence so does $\operatorname{Ker}\pi$. Since M is tilting, it also has a standard filtation. An inductive application of the Ext-vanishing statement in Lemma 4.3(b) thus shows that $\operatorname{Ext}^1_{\mathcal{G}(A)}(M, \operatorname{Ker}\pi) = 0$. Hence, applying $\operatorname{Hom}_{\mathcal{G}(A)}(M, -)$ to the exact sequence

$$0 \to \operatorname{Ker}\pi \to P(\lambda) \xrightarrow{\pi} \nabla(\lambda_{s-1}) \to 0$$

yields an exact sequence

$$0 \to \operatorname{Hom}_{\mathcal{G}(A)}(M, \operatorname{Ker}\pi) \to \operatorname{Hom}_{\mathcal{G}(A)}(M, P(\lambda)) \to \operatorname{Hom}_{\mathcal{G}(A)}(M, \nabla(\lambda_{s-1})) \to 0 \;.$$

In particular, there is $\nu\colon M \to P(\lambda)$ making the diagram

$$\begin{tikzcd} M \arrow[r, "\nu"] \arrow[d, two heads, "q"'] & P(\lambda) \arrow[dl, two heads, "\pi"] \\ \nabla(\lambda_{s-1}) & \end{tikzcd} \tag{69*}$$

commutative. From Diagrams (68*) and (69*) we obtain a commutative diagram

$$\begin{tikzcd} P(\lambda) \arrow[r, "\eta"] \arrow[dr, "\pi"'] & M \arrow[r, "\nu"] \arrow[d, "q"] & P(\lambda) \arrow[dl, "\pi"] \\ & \nabla(\lambda_{s-1}) \arrow[d, two heads] & \\ & \operatorname{Hd}\nabla(\lambda_{s-1}) = L(\lambda_{s-1}^{h^{-1}}) = L(\lambda) \;, & \end{tikzcd}$$

where the equation at the bottom follows from Theorem 5.8(f). The uniqueness of projective covers of $L(\lambda)$ now shows that $\nu \circ \eta$ is an isomorphism. In particular, ν is surjective and therefore $P(\lambda)$ is a direct summand of M. By induction on the length of the costandard filtration of M we obtain that M is in fact projective. $\square$

Here is a counter-example to the claim when A is *not* self-injective. We show at the beginning of the proof of Theorem 5.1 (as in the paper) that M is tilting if and only if $\operatorname{Res}_{B^-} M$ is projective and $\operatorname{Res}_{B^+} M$ is injective. Consider the non-selfinjective algebra $A \coloneqq \mathbb{C}[x_1, x_2]/(x_1^2, x_1x_2, x_2^2)$ with triangular decomposition $(A^- \coloneqq A, T \coloneqq \mathbb{C}, A^+ \coloneqq \mathbb{C})$. Then M is tilting if and only if M is projective. In particular, $M = A$ is tilting but it is not injective. If we swap the role of A^+ and A^- then the tilting modules are precisely the injective modules.

5. Cuspidal Calogero–Moser and Lusztig families for Coxeter groups[2]

There is a very tiny gap in the proof of Theorem 4.2 on page 216: $|Z(W)| = 1$ implies that $W \simeq G(m, m, n)$ **or** $W \simeq E_6$. We forgot the E_6 case. But here a quick

[2] J. Algebra 462 (2016), 197–252. With G. Bellamy.

computer check (for example in CHAMP using the function `ParabolicBranchingIndex`, or in CHEVIE with the respective commands) shows that every irreducible character of a proper parabolic of E_6 splits into at least two characters when inducing to E_6, so this case is fine too.

4. Restricted rational Cherednik algebras[3]

Some typos in appendix B: replace ρ by φ in equations (127) and (128), replace ζ by ζ^i in the definition of $\varphi_i(t)$ at the beginning of appendix B.

3. Decomposition matrices are generically trivial[4]

A typo in Theorem 1.21 (p. 14): Replace "$R \cap \mathcal{O} = \mathfrak{p}$" by "$R \cap \mathfrak{m} = \mathfrak{p}$".

In §6.2 I was not careful enough when defining constructible sets for *non-noetherian* schemes. Namely, a *constructible* subset of a (not necessarily noetherian) scheme X is defined to be a finite union of sets of the form $U \cap (X \setminus V)$ where $U, V \subseteq X$ are *retro-compact* open subsets. I forgot the retro-compactness. So, an arbitrary locally closed subset will in general not be constructible and not ind-constructible. For this to be true, we need to assume that X is noetherian. This is why in Proposition 6.2, Example 6.11, and Lemma 6.13 we need to assume that $\mathcal{P}$ is a property for algebras over *noetherian* rings. As we only apply these results for algebras over noetherian rings anyways, everything else remains valid.

2. CHAMP: A Cherednik Algebra Magma Package[5]

A typo in Theorem 1.4 (p. 272): I was sloppy writing down the R-basis of $R\langle V \oplus V^* \rangle$. Of course, it is not formed by elements of the form $\mathbf{x}_\alpha \mathbf{y}_\beta$. We need arbitrary products of x_i's and y_i's in arbitrary order. So, an R-basis is formed by elements of the form $(\mathbf{xy})_\gamma$ with $\gamma \in F_{2n}$, where $(\mathbf{xy})_\gamma = \prod_{i=1}^{2n} (\mathbf{xy})_{\gamma(i)}$ and $(\mathbf{xy})_{\gamma(i)} = x_i$ if $\gamma(i) \leq n$ and $(\mathbf{xy})_{\gamma(i)} = y_{\gamma(i)-n}$ otherwise. In the theorem "R-basis $\mathbf{x}_\alpha \mathbf{y}_\beta g$" should thus be replaced by "R-basis $((\mathbf{xy})_\gamma g)_{\gamma \in F_{2n}, g \in G}$". This does not change anything in the theorem, however.

[3] EMS Ser. Congr. Rep., Representation theory – current trends and perspectives (2016), 681–745.

[4] Int. Math. Res. Not. IMRN (2016), no. 7, 2157–2196.

[5] LMS J. Comput. Math. 18 (2015), no. 1, 266–307.