Algebraic Geometry WS 2024/2025
RPTU Kaiserslautern–Landau

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Exercise Sheet 12

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Exercises with numbers in brackets are taken from the book "An invitation to algebraic geometry" by Smith et. al. (2000).

Exercise 1 [7.5.1]**.** An algebraic variety is said to be *rational* if it is birationally equivalent to projective space $\mathbb{P}^n$ for some n. Show that the nodal plane curve $\mathbb{V}(y^2 - x^2 - x^3) \subset \mathbb{A}^2$ is rational.

Exercise 2. Show that morphisms from a *projective* variety to any quasi-projective variety are projective.

Exercise 3. Consider again the Whitney umbrella $V = \mathbb{V}(x^2 - y^2 z) \subset \mathbb{A}^3$. Recall that the singular locus of V is the whole z-axis. Compute the blowup of V along the z-axis and show that this is a desingularization of V.

Exercise 4. Consider again the higher order cusp $\mathbb{V}(y^2 - x^5) \subset \mathbb{A}^2$. Find an ideal $I \subset \mathbb{C}[X]$ such that the blowup $B_I(X) \to X$ is a desingularization.