Algorithmic Number Theory Prof. Dr. Ulrich Thiel
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Exercise Sheet 1

Oct 28, 2019

Please tell me in/after the lecture on Monday what you want to have discussed

Exercise 1. How many ways are there to write a pythagorean prime as a sum of two squares?

Exercise 2. Show that the ring $\mathbb{Z}[\sqrt{-5}] = \mathbb{Z} + \mathbb{Z}\sqrt{-5}$ is not factorial.

Exercise 3. Show that the ring $\mathbb{Z}[\sqrt{2}] = \mathbb{Z} + \mathbb{Z}\sqrt{2}$ is euclidean. Moreover, determine its units and its prime elements. Draw a picture of the splitting of primes as in the lecture for $\mathbb{Z}[i]$.

Hint: You need a norm function N as in the lecture for $\mathbb{Z}[i]$. In this case you could write it as $N(x) = x \cdot \overline{x}$. What is the conjugate of $\sqrt{2}$ (in the Galois sense)?

Exercise 4. For an integer $d > 1$, show that the ring $\mathbb{Z}[\sqrt{-d}] = \mathbb{Z} + \mathbb{Z}\sqrt{-d}$ only has ± 1 as units.

Exercise 5. For a square-free integer $d > 1$, show that the ring $\mathbb{Z}[\sqrt{d}] = \mathbb{Z} + \mathbb{Z}\sqrt{d}$ has infinitely many units.

Just for fun

Exercise 6. Write a program (e.g. in Python) to visualize the distribution of Gaussian primes in the complex plane.

Open problems

Exercise 7. There are infinitely many Gaussian primes along the real and along the imaginary axis. Are there any other lines in the complex plane containing infinitely many Gaussian primes?