Winter Term 20/21 *Prof. Dr. Ulrich Thiel*
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Commutative Algebra

Exercise Sheet 6

Due date: 8 December 2020, 9:00 am.

Exercise 1. Let $\varphi : A \to B$ be a ring morphism and let V and W be two A-modules. Show that there is a canonical isomorphism

$$(V \otimes_A W)^B \simeq V^B \otimes_B W^B$$

of B-modules.

Exercise 2. Let A be a local ring.[1] Let V and W be finitely generated A-modules. Show that if $V \otimes_A W = 0$, then already $V = 0$ or $W = 0$.

Exercise 3. Let A be a ring and let $f \in A$.

(a) Show that $\operatorname{Spec} A_f$ is homeomorphic to $\mathrm{D}(f) = \operatorname{Spec} A \setminus \mathrm{V}(f)$.

(b) Show that A_f is isomorphic to $A[X]/(fX - 1)$ as A-algebras.

Exercise 4. (a) Let A be a principal ideal domain and S a multiplicatively closed subset with $0 \notin S$. Show that $S^{-1}A$ is a principal ideal domain.

(b) Let K be a field and consider the polynomial ring $K[X]$ in one variable. The localization

$$K[X, X^{-1}] := K[X]_X = \{X\}^{-1}K[X]$$

is called the **Laurent polynomial ring** over K. Describe this ring explicitely by making sense of the notation $K[X, X^{-1}]$.

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[1] Recall that a ring is called local if it has a unique maximal ideal; see Exercise 2.3.