Algebraic Geometry WS 2024/2025 Prof. Dr. Ulrich Thiel
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Exercise Sheet $1\frac{1}{2}$

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Exercises with numbers in brackets are taken from the book "An invitation to algebraic geometry" by Smith et. al. (2000).

Exercise 1 [1.3.1]**.** Let $F\colon V \to W$ be a morphism of affine algebraic varieties. Prove that F is continuous in the Zariski topology.

Exercise 2

a. [1.3.2] Show that the twisted cubic V is isomorphic to the affine line by constructing an explicit isomorphism $\mathbb{A}^1 \to V$.

b. Recall that we can consider the general linear group $\mathrm{GL}(n, \mathbb{C})$ as an affine algebraic variety in $\mathbb{C}^{n^2+1}$. Show that the determinant $\det\colon \mathrm{GL}(n, \mathbb{C}) \to \mathbb{C}^\times$ is a morphism of affine algebraic varieties.

Exercise 3 [1.4.2]**.** Show that if $X \to Y$ is a surjective morphism of affine algebraic varieties, then the dimension of X is at least as large as the dimension of Y.

Exercise 4 [1.4.3]**.** Show that a hypersurface in $\mathbb{A}^n$ is irreducible if and only if the defining equation F is a power of an irreducible polynomial G.