Algebraic Geometry WS 2024/2025 | Prof. Dr. Ulrich Thiel
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Exercise Sheet 4

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Exercises with numbers in brackets are taken from the book "An invitation to algebraic geometry" by Smith et. al. (2000).

Exercise 1 [3.4.2]**.** Show that the homogeneous coordinate rings of projectively equivalent projective varieties are isomorphic.

Exercise 2 [3.4.3]**.** Find an example of two plane projective curves that are isomorphic but not projectively equivalent.

Exercise 3 [3.5.1]**.** Let $F, G \in \mathbb{C}[x, y, z]$ be two irreducible homogeneous quadratic polynomials. Show that there exists an automorphism of $\mathbb{P}^2$ mapping $\mathbb{V}(F)$ isomorphically onto $\mathbb{V}(G)$.
This shows that there exists only one (nondegenerate) projective conic up to a linear change of coordinates. Hint: An irreducible homogeneous quadratic polynomial defines a nondegenerate quadratic form on $\mathbb{C}^3$.

Exercise 4 [3.5.2]**.** Show that up to affine change of coordinates in the affine plane, there exists exactly two nonisomorphic affine (nondegenerate) conic plane curves. That is, the zero set of any irreducible quadratic polynomial $F \in \mathbb{C}[x, y]$ is – up to a linear change of coordinates – either a parabola $\mathbb{V}\left(y - x^2\right)$ or a hyperbola $\mathbb{V}(xy - 1)$, but the parabola and hyperbola are nonisomorphic.