Exercise Sheet 5

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Exercises with numbers in brackets are taken from the book "An invitation to algebraic geometry" by Smith et. al. (2000).

Exercise 1 [4.2.1]. Let V be an affine variety and f a function in its coordinate ring. Show that if f vanishes nowhere on V, then f is invertible in $\mathbb{C}[V]$.

Exercise 2 Show that regular functions on $\mathbb{P}^n$ are constant, i.e. $\mathcal{O}_{\mathbb{P}^n}(\mathbb{P}^n) = \mathbb{C}$.

Exercise 3 [4.3.1]. Show that the projection map pictured below[1] is regular.
(Hint: Choose (or change) coordinates for the plane so that the point p is the origin, l' is the y-axis, and l is the line $x = 1$.)
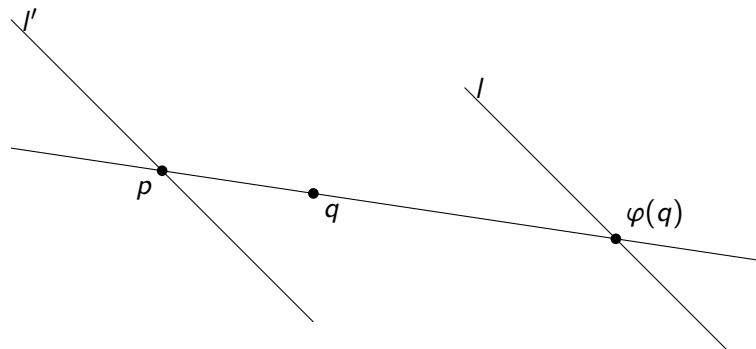
Exercise 4 [4.3.2]. Show that the ring $\mathcal{O}_V(U)$ of regular functions on the punctured plane $\mathbb{A}^2 - \{(0,0)\}$ is the polynomial ring $\mathbb{C}[x, y]$. Conclude that this quasi-projective variety is not affine.

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[1]See the bottom of page 57ff in "An Invitation to Algebraic Geometry" by Smith et al. (2000) for more details.