Exercise Sheet 11

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Exercises with numbers in brackets are taken from the book "An invitation to algebraic geometry" by Smith et. al. (2000).

Exercise 1. Show that the composition of projective morphisms is projective.

Exercise 2. Show that the blowup of the cusp $\mathbb{V}(y^2 - x^3) \subset \mathbb{A}^2$ in the origin is isomorphic to the twisted cubic curve.

Exercise 3. Consider the *Whitney umbrella* $V = \mathbb{V}(x^2 - y^2 z) \subset \mathbb{A}^3$.

1. Show that the singular locus of V is the whole z-axis.

2. Compute the blowup of V in the origin.

3. Argue why repeated blowups in points will not lead to a desingularization of V.

Exercise 4. Find out who introduced blowups. [I do not know! Zariski!?]

Exercise 5[7.3.4]**.** Prove that $\mathbb{P}^2$ is birationally equivalent, but not isomorphic, to $\mathbb{P}^1 \times \mathbb{P}^1$.

Exercise 6[7.3.3]**.** Prove that two irreducible quasi-projective varieties X and Y are birationally equivalent if and only if their function fields are isomorphic as $\mathbb{C}$-algebras.