Algebraic Geometry WS 2024/2025
RPTU Kaiserslautern–Landau

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In-Class Exercises 1

Tutorium on Oct. 22, 2024

Exercise 1. Revision of topological spaces.

- What is a topological space?
- What is the Hausdorff property?
- What is the closure and the interior of a set?
- What does it mean that a set is dense in a topological space?
- Show: a subset Y of a topological space X is dense if and only if every non-empty open subset of X intersects Y.
- What is the subspace topology?
- Revise the Euclidean, discrete, cofinite and product topology.

Exercise 2. Fundamental theorem of algebra.

- What is a polynomial with complex coefficients in N indeterminates?
- State the fundamental theorem of algebra.
- Use the fundamental theorem of algebra to classify the affine algebraic subvarieties of $\mathbb{C}$.
- Compare *polynomials* and *polynomial functions*.

Exercise 3. The real picture of a complex affine algebraic variety.

- Consider the affine algebraic variety $Z = \mathbb{V}(x^2 + y^2 - 1) \subset \mathbb{A}^2$. Show that Z is not bounded.
- Draw the real picture of Z. (Pretend that Z is a subvariety of $\mathbb{R}^2$ and treat its defining equation as polynomial with coefficients in $\mathbb{R}$.)
- Explain why the real picture of an affine algebraic variety can be misleading.