

## In-Class Exercises 2

Tutorium on Oct. 29, 2024

### Exercise 1. Irreducible components of a topological space

Revise section 2.7 and in particular proposition 2.7.7 of

<https://ulthiel.com/math/wp-content/uploads/lecture-notes/Commutative-Algebra.pdf>.

### Exercise 2. Prime, primary, maximal and radical ideals

1. What is an ideal in a commutative ring?
2. What does it mean that an ideal is finitely generated? What are principal ideals?
3. What are prime, primary and radical ideals?
4. What is a maximal ideal?
5. What are equivalent ways to characterize maximal and prime ideals?
6. Use OSCAR (<https://www.oscar-system.org/>) to play with these notions:
  - Create polynomial rings and ideals.
  - Check if an ideal is prime, primary, radical or maximal.
  - Quotient a polynomial ring by an ideal.
  - Factor a multivariate polynomial.

### Exercise 3. Noetherian rings

1. What is a Noetherian ring?
2. Is  $\mathbb{C}[x_1, \dots, x_n]$  a Noetherian ring? (Cf. Hilbert's basis theorem.)

### Exercise 4. Unique factorization domains

1. What is an integral domain?
2. What is an irreducible element of an integral domain?
3. What is a unique factorization domain?