Algebraic Geometry WS 2023/2024
RPTU Kaiserslautern–Landau

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Exercise Sheet 2

Release: November 2, 2023
Deadline: **November 9, 2023 by 10:00 a.m. Kaiserslautern**

Each exercise is worth 4 points. You need a minimum of 50% of the total points of all exercise sheets by the end of the semester in order to obtain the "Schein". Submit your solutions via OLAT by uploading **one** pdf-file with all your solutions **before 10:00 a.m.** Kaiserslautern.

You may submit your solution individually or in a group of at most 2 people. If you opt for a group submission, state the names of both individuals on the first page of the submitted pdf-file.

Exercises with numbers in brackets are taken from the book "An invitation to algebraic geometry" by Smith et. al. (2000).

Exercise 1 [1.3.1]**.** Let $F\colon V \to W$ be a morphism of affine algebraic varieties. Prove that F is continuous in the Zariski topology.

Exercise 2

a. [1.3.2] Show that the twisted cubic V is isomorphic to the affine line by constructing an explicit isomorphism $\mathbb{A}^1 \to V$.

b. Recall that we can consider the general linear group $\mathrm{GL}(n, \mathbb{C})$ as an affine algebraic variety in $\mathbb{C}^{n^2+1}$. Show that the determinant $\det\colon \mathrm{GL}(n, \mathbb{C}) \to \mathbb{C}^{\times}$ is a morphism of affine algebraic varieties.

Exercise 3 [1.4.2]**.** Show that if $X \to Y$ is a surjective morphism of affine algebraic varieties, then the dimension of X is at least as large as the dimension of Y.

Exercise 4 [1.4.3]**.** Show that a hypersurface in $\mathbb{A}^n$ is irreducible if and only if the defining equation F is a power of an irreducible polynomial G.