Algebraic Geometry WS 2023/2024 | Prof. Dr. Ulrich Thiel
RPTU Kaiserslautern–Landau | Dr. Martin Bies

# Exercise Sheet 5

Release: December 7, 2023
Deadline: **December 14, 2023 by 10:00 a.m. Kaiserslautern**

Each exercise is worth 4 points. You need a minimum of 50% of the total points of all exercise sheets by the end of the semester in order to obtain the "Schein". Submit your solutions via OLAT by uploading **one** pdf-file with all your solutions **before 10:00 a.m.** Kaiserslautern.

You may submit your solution individually or in a group of at most 2 people. If you opt for a group submission, state the names of both individuals on the first page of the submitted pdf-file.

Exercises with numbers in brackets are taken from the book "An invitation to algebraic geometry" by Smith et. al. (2000).

**Exercise 1** [3.4.2]**.** Show that the homogeneous coordinate rings of projectively equivalent projective varieties are isomorphic.

**Exercise 2** [3.4.3]**.** Find an example of two plane projective curves that are isomorphic but not projectively equivalent.

**Exercise 3** [3.5.1]**.** Let $F, G \in \mathbb{C}[x, y, z]$ be two irreducible homogeneous quadratic polynomials. Show that there exists an automorphism of $\mathbb{P}^2$ mapping $\mathbb{V}(F)$ isomorphically onto $\mathbb{V}(G)$.
This shows that there exists only one (nondegenerate) projective conic up to a linear change of coordinates. Hint: An irreducible homogeneous quadratic polynomial defines a nondegenerate quadratic form on $\mathbb{C}^3$.

**Exercise 4** [3.5.2]**.** Show that up to affine change of coordinates in the affine plane, there exists exactly two nonisomorphic affine (nondegenerate) conic plane curves. That is, the zero set of any irreducible quadratic polynoial $F \in \mathbb{C}[x, y]$ is – up to a linear change of coordinates – either a parabola $\mathbb{V}\left(y - x^2\right)$ or a hyperbola $\mathbb{V}(xy - 1)$, but the parabola and hyperbola are nonisomorphic.

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Live music, waffles, lentil soup, and mulled wine!

# FS-Mathe Christmas Party

Thursday, the 14th of December
6:00 pm
Building 48, 5th Floor