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Exercise Sheet 6

Release: December 14, 2023
Deadline: **December 21, 2023 by 10:00 a.m. Kaiserslautern**

Each exercise is worth 4 points. You need a minimum of 50% of the total points of all exercise sheets by the end of the semester in order to obtain the "Schein". Submit your solutions via OLAT by uploading **one** pdf-file with all your solutions **before 10:00 a.m.** Kaiserslautern.

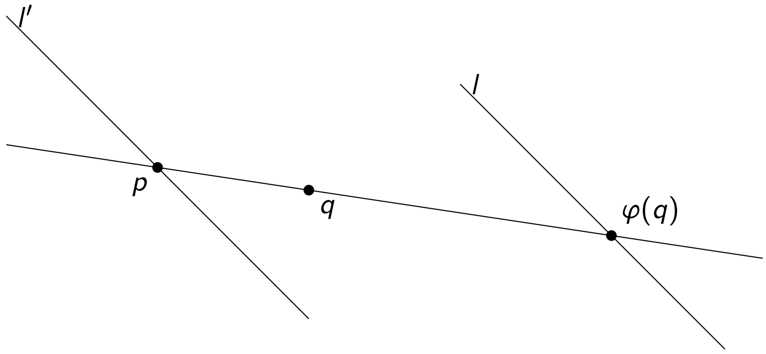
You may submit your solution individually or in a group of at most 2 people. If you opt for a group submission, state the names of both individuals on the first page of the submitted pdf-file.

Exercises with numbers in brackets are taken from the book "An invitation to algebraic geometry" by Smith et. al. (2000).

Exercise 1 [4.2.1]**.** Let V be an affine variety and f a function in its coordinate ring. Show that if f vanishes nowhere on V, then f is invertible in $\mathbb{C}[V]$.

Exercise 2 Show that regular functions on $\mathbb{P}^n$ are constant, i.e. $\mathcal{O}_{\mathbb{P}^n}(\mathbb{P}^n) = \mathbb{C}$.

Exercise 3 [4.3.1]**.** Show that the projection map pictured below[1] is regular.
(Hint: Choose (or change) coordinates for the plane so that the point p is the origin, l' is the y-axis, and l is the line $x = 1$.)



Exercise 4 [4.3.2]**.** Show that the ring $\mathcal{O}_V(U)$ of regular functions on the punctured plane $\mathbb{A}^2 - \{(0,0)\}$ is the polynomial ring $\mathbb{C}[x, y]$. Conclude that this quasi-projective variety is not affine.

[1] See the bottom of page 57ff in "An Invitation to Algebraic Geometry" by Smith et al. (2000) for more details.