

Exercise Sheet 7

Release: December 21, 2023

Deadline: **January 11, 2024 by 10:00 a.m. Kaiserslautern**

Each exercise is worth 4 points. You need a minimum of 50% of the total points of all exercise sheets by the end of the semester in order to obtain the “Schein”. Submit your solutions via OLAT by uploading **one** pdf-file with all your solutions **before 10:00 a.m.** Kaiserslautern.

You may submit your solution individually or in a group of at most 2 people. If you opt for a group submission, state the names of both individuals on the first page of the submitted pdf-file.

Exercises with numbers in brackets are taken from the book “An invitation to algebraic geometry” by Smith et. al. (2000).

Exercise 1 [5.1.1].

Consider the Veronese map $\mathbb{P}^2 \xrightarrow{\nu_2} \mathbb{P}^5$. Its image is called the *Veronese surface*. Describe the images of the lines in \mathbb{P}^2 on the Veronese surface.

Exercise 2 [5.2.1].

Given five points in \mathbb{P}^2 as above, prove that there is a *unique* conic passing through these points unless four of the points lie on a line. Show that the conic is nondegenerate unless three of the points lie on a line.

Exercise 3 [5.2.3].

Given four points and a line in \mathbb{P}^2 , show that typically two conics pass through the four points and are tangent to the line. Under what special conditions on the position of the points and the line do we fail to get exactly two?

Hint: As we will discuss in detail in Section 6.1, a line is tangent to a conic if the defining quadratic function has a double root when restricted to the line; on the other hand, a quadratic polynomial has a double root if and only if its discriminant, a degree two polynomial in its coefficients, is zero.

Exercise 4 [5.2.4].

- How many conics in \mathbb{P}^2 do we expect through 3 given points and tangent to 2 given lines?
- How many conics in \mathbb{P}^2 do we expect through 2 given points and tangent to 3 given lines?
- How many conics in \mathbb{P}^2 do we expect tangent to 5 given lines?

Merry Christmas and a Happy New Year!