

## Exercise Sheet 10

Release: January 25, 2024

Deadline: **February 1, 2024 by 10:00 a.m. Kaiserslautern**

Each exercise is worth 4 points. You need a minimum of 50% of the total points of all exercise sheets by the end of the semester in order to obtain the “Schein”. Submit your solutions via OLAT by uploading **one** pdf-file with all your solutions **before 10:00 a.m.** Kaiserslautern.

You may submit your solution individually or in a group of at most 2 people. If you opt for a group submission, state the names of both individuals on the first page of the submitted pdf-file.

Exercises with numbers in brackets are taken from the book “An invitation to algebraic geometry” by Smith et. al. (2000).

**Exercise 1** [5.5.2]. Find an example of two plane projective curves that are isomorphic but have different degrees.

**Exercise 2** [5.5.3]. Find an example of two plane projective curves that have the same degree but are not isomorphic.

**Exercise 3** [5.6.1]. Assume that the variety  $V \subset \mathbb{P}^n$  has the Hilbert polynomial  $P$ . Calculate the Hilbert polynomial of the image variety  $\nu_d(V) \subset \mathbb{P}^{\binom{n+d}{d}-1}$  under the degree- $d$  Veronese map.

**Exercise 4.** Suppose  $V = \{p_1, p_2, p_3\} \subset \mathbb{P}^2$  consists of three points. Compute the Hilbert function of  $V$ .