

In-Class Exercises 1

Tutorium on Oct. 24, 2023

Exercise 1. Revision of topological spaces.

- What is a topological space?
- What is the Hausdorff property?
- What is the closure and the interior of a set?
- What does it mean that a set is dense in a topological space?
- Revise the Euclidean, discrete, cofinite and product topology.

Exercise 2. Fundamental theorem of algebra.

- What is a polynomial with complex coefficients in N indeterminates?
- State the fundamental theorem of algebra.
- Use the fundamental theorem of algebra to classify the affine algebraic subvarieties of \mathbb{C} .

Exercise 3. The real picture of a complex affine algebraic variety.

- Consider the affine algebraic variety $Z = V(x^2 - y^2 - 1) \subset \mathbb{A}^2$. Show that Z is not bounded.
- Draw the real picture of Z . (Pretend that Z is a subvariety of \mathbb{R}^2 and treat its defining equation as polynomial with coefficients in \mathbb{R} .)
- Explain why the real picture of an affine algebraic variety can be misleading.