In-Class Exercises 2

Tutorium on Oct. 31, 2023

Exercise 1. Irreducible components of a topological space
Revise section 2.7 and in particular proposition 2.7.7 of

> https://ulthiel.com/math/wp-content/uploads/lecture-notes/Commutative-Algebra.pdf.

Exercise 2. Prime, primary, maximal and radical ideals

1. What is an ideal in a commutative ring?

2. What does it mean that an ideal is finitely generated? What are principal ideals?

3. What are prime, primary and radical ideals?

4. What is a maximal ideal?

5. What are equivalent ways to characterize maximal and prime ideals?

6. Use OSCAR (https://www.oscar-system.org/) to play with these notions:

   - Create polynomial rings and ideals.
   - Check if an ideal is prime, primary, radical or maximal.
   - Quotient a polynomial ring by an ideal.
   - Factor a multivariate polynomial.

Exercise 3. Noetherian rings

1. What is a Noetherian ring?

2. Is $\mathbb{C}[x_1, \ldots, x_n]$ a Noetherian ring? (Cf. Hilbert's basis theorem.)

Exercise 4. Unique factorization domains

1. What is an integral domain?

2. What is an irreducible element of an integral domain?

3. What is a unique factorization domain?