Character theory of finite groups — **Exercise Sheet 5**
RPTU Kaiserslautern-Landau — **FB Mathematik**

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Please submit your solution alone or with a (one!) partner in the postbox "Character Theory" on the ground floor, in the office of Dr. Tobias Metzlaff (48-424) or by mail at metzlaff@mathematik.uni-kl.de.

Exercises with numbers in brackets are taken from the book "Introduction to Representation Theory" by Pavel Etingof et al from 2011 (https://math.mit.edu/~etingof/reprbook.pdf).

Exercise 16 (4.12.1)
The dihedral group $\mathfrak{D}_N$ of order $2N$ is generated by two elements r, s with $r^N = s^2 = srsr = 1$. Describe all the complex irreducible representations of $\mathfrak{D}_N$ for $N \geq 2$ even.
(*Hint*: There are four irreducible one–dimensional representations and $N/2 - 1$ two–dimensional ones. For the latter, write s and r as matrices of a reflection and a rotation.)

Exercise 17
Let G be a finite group and V be a complex irreducible representation with a G–invariant bilinear form $B: V \times V \to \mathbb{C}$. Show the following statements.

(1) If B is symmetric or anti–symmetric, then it is also nondegenerate.

(2) If B is anti–symmetric, then the dimension of V is even.

(*Remark*: (1) appears in the proof of Theorem 5.1.5, (2) is mentioned after Definition 5.1.1.)

Exercise 18 (5.1.2)
Let G be a finite group and V be a complex irreducible representation. Show that V is of real type if and only if V is of the form $W \otimes_{\mathbb{R}} \mathbb{C}$ for some real representation W.
(*Remark*: $W \otimes_{\mathbb{R}} \mathbb{C}$ is called the complexification of W.)

Exercise 19 (5.1.7)
Let G be a finite nontrivial group of odd order. Show that there is a complex irreducible representation, which is NOT the complexification of a real representation.