CHARACTER THEORY OF FINITE GROUPS | **EXERCISE SHEET 7**
RPTU KAISERSLAUTERN-LANDAU | **FB MATHEMATIK**

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Due date: Thursday, 25.07.2024, 12:00
SS 2024

Please submit your solution alone or with a (one!) partner in the postbox "Character Theory" on the ground floor, in the office of Dr. Tobias Metzlaff (48-424) or by mail at metzlaff@mathematik.uni-kl.de.

Exercises with numbers in brackets are taken from the book "Introduction to Representation Theory" by Pavel Etingof et al from 2011 (https://math.mit.edu/~etingof/reprbook.pdf).

EXERCISE 24
Let G be a finite simple group. Show that G is solvable if and only if G is Abelian.

EXERCISE 25
Classify the finite simple Abelian groups up to isomorphism.

EXERCISE 26 (5.8.4)
Let $K \subseteq H \subseteq G$ be groups and V be a representation of G. Show that $\mathrm{Ind}_H^G \mathrm{Ind}_K^H V$ and $\mathrm{Ind}_K^G V$ are isomorphic as representations of G.

EXERCISE 27 (5.8.5)
Let $H \subseteq G$ be finite groups and $\chi : H \to \mathbb{C}^*$ be a homomorphism. Denote the corresponding complex one-dimensional representation of H by $\mathbb{C}_\chi$ and let

$$e_\chi = \frac{1}{|H|} \sum_{g \in H} \chi(g)^{-1} g \in \mathbb{C}[H]$$

be the idempotent corresponding to χ. Show that the G-representation $\mathrm{Ind}_H^G \mathbb{C}_\chi$ is naturally isomorphic to $\mathbb{C}[G]\, e_\chi$, where G acts on the latter by left-multiplication.