Algebraic Geometry (WS 2024/2025)

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Welcome to the course "Algebraic Geometry" in the winter term 2024/2025!

What this course is about

Algebraic geometry is the study of zero sets of (systems of) polynomials.

> Algebraic geometry occupies a central place in modern mathematics and has multiple conceptual connections with such diverse fields as complex analysis, topology and number theory. As a study of systems of polynomial equations in several variables, the subject of algebraic geometry begins with finding specific solutions via equation solving, and then proceeds to understand the intrinsic properties of the totality of solutions of a system of equations. This understanding requires both conceptual theory and computational technique. — Wikipedia, https://en.wikipedia.org/wiki/Algebraic_geometry

What you will learn

You will learn about the interplay between algebra and geometry. You will learn basic notions like affine and projective varieties and their morphisms; properties like dimension, smoothness, and degree; classical constructions like the Veronese map and the Grassmannian; examples like rational normal curves and Segre varieties; you will get a glimpse of the birational classification and of the role of vector bundles.

What we will do

We will follow a nice little book titled "An Invitation to Algebraic Geometry" [2] by K. Smith et. al. (2000). This is available for free via our library at

https://hbz-rptu.primo.exlibrisgroup.com/permalink/49HBZ_RTU/1fh1ebs/cdi_askewsholts_vlebooks_9781475744972

The plan is to cover the whole book. I have (hand-)written up my own version with more colors but basically the exact same content at:

https://ulthiel.com/math/teaching/notes-repository/

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[*]https://ulthiel.com/math

The password will be given in class. My notes currently only go up to Section 7.1. I will complete them later during the semester.

Formalities

This module is worth **9 CP**. More information is in the module handbook https://modhb.rptu.de/mhb/modules/MAT-40-12-M-7/.

The lectures are on **Tuesdays 10:00–11:30** and **Thursdays 10:00–11:30** in **48-438**, starting on October 22, 2024.

Additionally, we have exercise sessions on **Tuesdays 15:30–17:00** in **48-438**, starting from October 22, 2024. This semester I will take care of the exercise sessions by myself. An exercise sheet will be released each Tuesday

Note: We will really have an exercise session already on October 22: I have prepared in-class exercises!

Flashcards

Prerequisites

For experts

For those who have basic knowledge of algebraic geometry already or who need more material per time, I recommend you read the book "Algebraic Geometry. A first course." [1] by J. Harris (1992) in parallel! This book is available for free via our library at

https://hbz-rptu.primo.exlibrisgroup.com/permalink/49HBZ_RTU/1fh1ebs/cdi_askewsholts_vlebooks_9781475721898

References

[1] Joe Harris. *Algebraic geometry*. Vol. 133. Graduate Texts in Mathematics. A first course. Springer-Verlag, New York, 1992, pp. xx+328. DOI: 10.1007/978-1-4757-2189-8. URL: https://doi.org/10.1007/978-1-4757-2189-8.

[2] Karen E. Smith, Lauri Kahanpää, Pekka Kekäläinen, and William Traves. *An invitation to algebraic geometry*. Universitext. Springer-Verlag, New York, 2000, pp. xii+155. DOI: 10.1007/978-1-4757-4497-2. URL: https://doi.org/10.1007/978-1-4757-4497-2.