Geometry of Schemes (SS2024)

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Welcome to the course "Geometry of Schemes" in the summer term 2024!

Affine schemes are geometric spaces which can be constructed for arbitrary commutative rings, extending the classical correspondence between affine algebraic varieties and finite type algebras over an algebraically closed field. Schemes are the fundamental spaces in modern algebraic geometry and since they are so general, they have applications in many other fields, for example number theory and representation theory.

In this course, you will learn the basics of scheme theory. We will follow a nice book titled "The Geometry of Schemes" [1] by David Eisenbud & Joe Harris (2000). This is available for free via our library at

https://hbz-rptu.primo.exlibrisgroup.com/permalink/49HBZ_RTU/1v7fsfl/alma990367074540206441

The plan is to cover most of the material of the first two chapters (pages 1–90).

The lectures are on **Thursdays, 11:45–13:15 in 48-538**, starting on April 25, 2024. There are two exceptions: May 9, 2024 (public holiday); May 30, 2024 (public holiday). The target pace is thus about **8 pages per week**.

> I want to emphasize one important aspect. According to the module handbook[a] the effort for this course is 4.5 CP. This amounts to 135h of time, split into 28h presence time (the lectures) and **107h self-study time**. Note that the self-study time is almost **four times** as much as the presence time! I will require you to work these hours additionally to the lecture! Quoting David P. Roberts: "My job is to make your learning effort as efficient and pleasant as possible, but it is your job to put in the quality time!"
>
> [a] https://modhb.uni-kl.de/mhb/modules/MAT-42-13-M-7/

I will keep a "lecture notebook" below.

Prerequisites

This is an advanced course. You need to be fluent in the basics of **commutative algebra**. Experience with classical algebraic geometry over algebraically closed fields is theoretically not necessary but strongly recommended. This course is not accompanied by exercise sessions. Exercises are spread throughout the text in the book. I strongly recommend doing every single of these exercises!

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Hint

For annotating the book (especially with your questions) you can transform the PDF to have larger margins with the tool `pdf-crop-margins`[1] as follows:

```
$ pdf-crop-margins -o out.pdf -u -s -p 100 -a4 0 0 -300 0 book.pdf
```

For annotating PDFs, I recommend the cross-platform open-source software Xournal++[2].

Lecture logbook

Lecture 1 (April 25, 2024)

Pages 1–13.5.

Homework

1. Review sheaves.

Lecture 2 (May 2, 2024)

Pages 13.5–21.

Homework

1. I forgot to mention the pushforward of sheaves, top of page 18. This is very simple but please study this.
2. To practice sheaves, you should do all the exercises in this section of the book.
3. For some examples and counterexamples in sheaf theory see my notes on my website (in Teaching > Notes repository).

References

[1] David Eisenbud and Joe Harris. *The geometry of schemes*. Vol. 197. Graduate Texts in Mathematics. Springer-Verlag, New York, 2000, pp. x+294.

[1] https://pypi.org/project/pdfCropMargins/
[2] https://xournalpp.github.io/